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An Improved Statistical Approach for Soft Computing

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Abstract

Soft computing is a fusion of methodologies that are designed to obtain working solutions to real world problems quickly accepting approximations and unconventional approaches. Its main aim is to exploit the tolerance for the imprecision, uncertainty, approximate reasoning and partial truth in order to achieve the close resemblance with human like decision making. Statistics is more rigorous and focuses on establishing objective conclusions based on experimental data by analyzing the possible situations and their likelihood. It emphasizes the need for mathematical methods and tools to assess solutions and guarantee performance. Combining the two fields enhance the robustness and generalizability of data analysis methods, while preserving the flexibility to solve real-world problems efficiently and intuitively.

This paper modifies the idea of soft probability, where a model for computing probabilities of fuzzy systems and events is constructed.

Keywords: *Fuzzy Set, Fuzzy computing, Probabilistic computing, Data Analysis, Soft Computing, Statistics.*

Introduction

Soft Computing is an emerging approach to the computing that is designed to model and enable solutions to the real world problems, which are not modeled or too difficult to model mathematically. The idea of soft computing was initiated by Dr. Lotfi Zadeh[1981]. Dr. Zadeh defined soft computing in its latest Genetic computing and Probabilistic computing into one multidisciplinary system. The application of soft computing has proved two main advantages. First, it help in solving nonlinear problems, in which mathematical models are not available. Second, it introduced the human knowledge such as cognition, recognition, understanding learning and others into the fields of computing. Soft computing consists of several computing paradigms like Fuzzy sets, rough sets, Neural Networks, Genetic Algorithms Evolutionary computing, Probabilistic and evidential reasoning, Multi-valued logic and related fields [Kacpzyk 2001]. Other scientists [Dote et.al 2000] proposed the notion of extended soft Computing [ESC] as anew discipline developed by adding Chaos computing and Immune network Theory to the Classical Soft Computing (CSC), as defined and proposed by Lotfi Zadeh.

The invention, or proposition, of the Fuzzy sets was motivated by need to capture and represent the real world with its fuzzy data due to uncertainty, which may be caused by imprecision in the measurement due to imprecision of tools or other factors. Uncertainty can also be caused by the vagueness in the language. Dr. Zadeh realized that existing crisp sets do not provide adequate representation for most cases. So, instead of avoiding or ignoring uncertainty, Dr. Zadeh developed a set theory that capture this uncertainty. So, rather defining this crisp set with absolute uncertainty, Dr. Zadeh proposed the concept of membership function. An element can be in the set with a degree of membership. Hence, Crisp sets are a special case, or a subset, of fuzzy sets, where elements are allowed a membership degree of 100% or 0% but nothing in between.

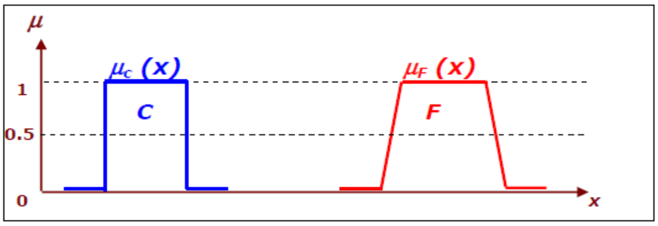

Fig 1: Membership function of crisp set C and Fuzzy SetF

The origin of the probabilistic reasoning dates back to the eighteenth century. One of the two major paradigms of probabilistic reasoning is called Bayesian Belief Networks. This paradigm is based on the work of Thomas Bayes[Bayes 1763]. The other is called Dampster's Shafers theory'(DST) of Belief, also known as the Mathematical Theory of evidence(MTE) which was developed by Dempster and Shafer [Dempster1967], Shafer[1976].Probability, as a measure of the belief, does not always involve future prediction. Consider the following statements for example, “It is 90% probable that Shakespeare actually wrote Hamlet” and “the probability that Oswald acted alone in assassinating Kennedy is 0.8”[Ross

1994]. Here we notice that uncertainty is not the result of fuzziness as partial truth is not being considered here.

Both the theories probability theory and Fuzzy set theory deals with uncertainty. The uncertainty described by probability is randomness while the uncertainty described by fuzzy set theory is known as Fuzziness. Probability theory deals with the prediction of a future event based on the information currently available. On the other hand fuzzy set theory deals with the concept and status of variables rather than events [Klir and et. al. 1997]. In 1968, Lotfi Zadeh published a paper on computing the probability of fuzzy events. He recalls classical probability of event A as

$P(A) = E(\mu_A)$

Where A= Fuzzy event, μ_A= characteristic function of A(0 or 1 in traditional probability)

and

$$E(\mu_A) = \int_{R^n} \mu_A(x)dP = P(A)$$

Dr. Zadeh replaced the Boolean characteristic function by generalized function

$$\mu_A : R^n \to [0,1]$$

$$P(A) = \sum_{x_j \in X} P(x_i)\,\mu_A(x_i)$$

I.Success Criteria

Before presenting the proposed model, it is important to determine the set of criteria for a successful proposed model. So, our propose is to fulfill the following objectives:

1. Since probability theory has been proven successful in the use of crisp computation, then an important criterion of the success of any proposed model is to be reducible to the axioms of the current theory when Fuzziness reduces to crisp sets, recalling that crisp sets are a special case of fuzzy sets.
2. The proposed model should be computationally feasible.
3. The proposed model should offer better probability estimation.
4. Since the computational gains obtained from the use of fuzzy set theory are highly dependent on the choice of fuzzy membership function and the partitioning scheme. He desired that the probability theory should be sensitive to these two factors.
5. It is intuitive that higher probability should be obtained from higher frequency, than from lower frequency of comparable memberships in the events.
6. It is intuitive that higher probability should be obtained from higher membership values than from lower membership values of comparable frequencies, the higher the membership the stronger the positiveness about the occurrence of the event.
7. The fundamental axioms of probability should always be preserved, that is:

$$0 \leq P(E) \leq 1.$$

III. Proposed Model

Before proposing the model, we define P (E), the probability of a simple event E. classically, P (E) is defined as

$$P(E) = \lim_{\substack{n\to\infty \\ N\to\infty}} \frac{n(E)}{N} \quad \ldots\ldots\ldots\ldots (1)$$

In a similar framework, Philippe Smets extended the theory of belief functions by defining the belief of a fuzzy event [Smets, 1981]. Neither of the above papers did approach the fundamental problem of calculating the probability of simple , atomic, fuzzy events, which was later on discovered by Moussa,[Moussa,2003]. He replaces the traditional probability with soft probability to process soft system computations. According to Moussa [2003] any partial occurrence of E is a Boolean variable either E occurred, increment 'n' and 'N' or E did not occur, increment only.

$$P(E) = \lim_{\substack{i\to\infty \\ n\to\infty \\ N\to\infty}} \sum \frac{n_i\ \mu_{Ei}}{N} \quad \ldots\ldots\ldots\ldots (2)$$

where 'n_i' is the number of times E occurred with a membership degree of μ_{Ei} in N repetitions.

Arora et.al. [2013] worked on the above model and gave an equation that was more towards the precision.

$$P(E) = \lim_{\substack{i\to\infty \\ n\to\infty \\ N\to\infty}} \sum \frac{n_i(\log n_i)\mu_{E_i}}{N} \quad \ldots\ldots\ldots (3)$$

Where n_i is number of times E occurred with a membership degrees of μ_E, in N repetition of experiment. Here $(\log n_i)$ factor is used to increase precision in the earlier existing soft probabilities proposed by Moussa [2003].

Now instead of multiplying factor $(\log n_i)$ if we add this factor then we obtain a new expression for the Soft Probability i.e.

$$P(E) = \lim_{\substack{i\to\infty \\ n\to\infty \\ N\to\infty}} \sum \frac{n_i \mu_{E_i} + (\log n_i)}{N} \quad \ldots\ldots\ldots (4)$$

So, probability stated in equation (4) is called as Improved Soft Probability (ISP).

Suppose we have an experiment in which the outcome could be any positive real number. Let us suppose further that we are only interested in the first few number which are partitioned into 4 classes, or sets. We take the class size to be 4 units. For simplicity and without the loss of generality we assume that we ran the experiment, say 23 times and that the outcome was 3.25 five times, 3.8 six times and 5 twelve times. Figure 2 illustrate the example with crisp tangent classes.

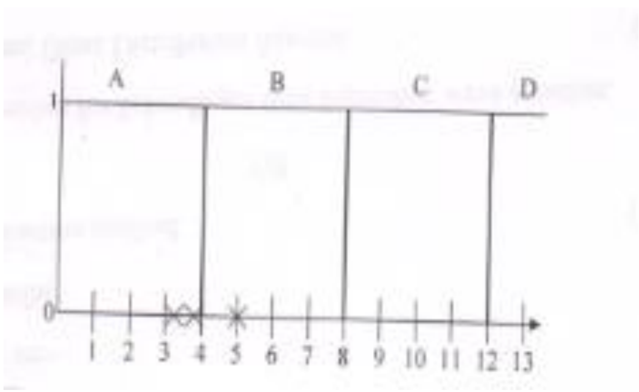

Fig2: Computing probabilities with crisp tangent classes

Now let us compute the probability of the outcome being in class B, P (B) & the probability of the outcome being in A, P (A).

Using equation (1)

$$P(B) = \frac{12}{23} \approx 0.52 \ and \quad P(A) = \frac{11}{23} \approx 0.48$$

Soft probability by Moussa using equation (2) be

$$P(B) = \frac{(5*0) + (6*0) + (2*1)}{23} \approx 0.52$$

and

$$P(A) = \frac{(5*1) + (6*1) + (12*0)}{23} \approx 0.48$$

Modified Soft Probability (MSP) using the equation (3) be

$$P(B) = \frac{(5*0)(\log 5) + (6*0)(\log 6) + (12*1)(\log 12)}{23} \approx 0.6$$

$$P(A) = \frac{(5*1)(\log 5) + (6*1)(\log 6) + (12*0)(\log 12)}{23} \approx 0.4$$

Now the Improved Soft Probability (ISP) using the equation (4) be

$$P(B) = \frac{[(5*0) + \log 5] + [(6*0) + \log 6] + [(12*1) + \log 12]}{23} \approx 0.6$$

$$P(A) = \frac{[(5*1) + \log 5] + [(6*1) + \log 6] + [(12*0) + \log 12]}{23} \approx 0.4$$

From above it is clear that CPE - classical probability equation (1), SPE - Soft Probability equation (2), MSP equation (3) yield the approx. same value and the ISP gives us the much better results, i.e. Improved Soft Probabilitiy (ISP) is the most precise one. Now we change the class positioning into the fuzzy overlapping classes and using the same class sizes. We use the trapezoidal membership function as in fig.3

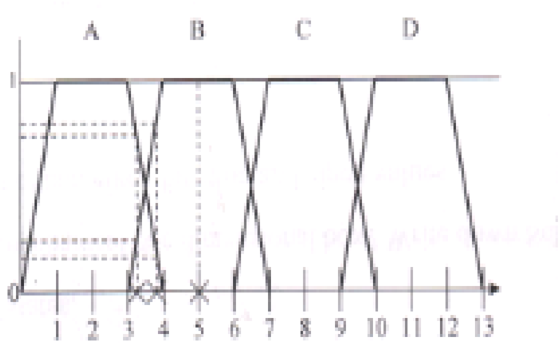

Fig.3: Computing Probabilities with fuzzy overlapping classes

Now we compute the probabilities of the same events, A and B. if we try to do the computation using eq.(1) we have to first make a decision on what counts as an occurrence of the event and what does not. For instance there is no question about the outcome of 5 being in class B. but, when the outcome of system is 3.25, the membership function specifies this as being in class A with 75% certainty and in class B with25% certainty. We obviously do not have a mechanism of defining what constitutes an occurrence of the system output in a class according to equation(1) and consequently , we cannot perform the computation of the probability of an event that is not defined. However, using equation (2) & (3) computation is straight forward.

Using equation (2)

$$P(B) = \frac{(5 * 0.25) + (6 * 0.8) + (12 * 1)}{23} \approx 0.8$$

$$P(A) = \frac{(5 * 0.75) + (6 * 0.18) + (12 * 0)}{23} \approx 0.2$$

Using equation (3)

$$P(B) = \frac{(5 * 0.25)\log 5 + (6 * 0.8)\log 6 + (12 * 1)\log 12}{23} \approx 0.8$$

$$P(A) = \frac{(5 * 0.75)\log 5 + (6 * 0.18)\log 6 + (12 * 0)\log 12}{23} \sim 0.2$$

And by using the equation (4)

$$P(B) = \frac{[(5 * 0.25) + \log 5] + [(6 * 0.8) + \log 6] + [(12 * 1) + \log 12]}{23} \approx 0.8$$

$$P(A) = \frac{[(5 * 0.75) + \log 5] + [(6 * 0.18) + \log 6] + [(12 * 0) + \log 12]}{23} \sim 0.2$$

By studying the example above we come to know that the proposed Improved Soft Probability (ISP) meets the criteria stipulated for a successful model in the previous section.

IV. Comparative Study

All the existing methods provide the same results for Crisp Tangent classes while the ISP provides a more accurate probability. But in Fuzzy overlapping of classes all the existing methods and the ISP provides the same results.

V. Conclusion

Fuzzy computing and probabilistic computing are powerful tools in the weapon store of soft computing. Present Research in this paper aims at developing an improved model to provide more precise, significant, computationally feasible probability. It satisfies all the success criteria laid above.

In other words, we can ISP significantly improves the precision power of probabilistic soft computing.

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